HARSHBARGER • REYNOLDS

Mathematical Applications

for the Management, Life, and Social Sciences

11TH EDITION

Index of Selected Applications

Business and Management

Economics

Finance

Life Science

Mathematical Applications

for the Management, Life, and Social Sciences

11th Edition

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***Mathematic Applications for the Management, Life, and Social Sciences*, Eleventh Edition**

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Cover Image: Sölvesborg Bridge in Sweden has been named one of the ten most interesting bridges in the world in 2013—a nomination shared with a series of spectacular bridges, including in New Zealand, Austria and China. Mathematics is itself a bridge that links several disciplines together to translate the architect's conception of the Sölvesborg Bridge to its physical reality.

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Preface

To paraphrase English mathematician, philosopher, and educator Alfred North Whitehead, the purpose of education is not to fill a vessel but to kindle a fire. In particular, Whitehead encouraged students to be creative and imaginative in their learning and to continually form ideas into new and more exciting combinations. This desirable goal is not always an easy one to realize in mathematics with students whose primary interests are in areas other than mathematics. The purpose of this text, then, is to present mathematical skills and concepts and to apply them to ideas that are important to students in the management, life, and social sciences. We hope that this look at the relevance of mathematical ideas to a broad range of fields will help inspire the imaginative thinking and excitement for learning that Whitehead spoke of. The applications included allow students to view mathematics in a practical setting relevant to their intended careers. Almost every chapter of this book includes a section or two devoted to the applications of mathematical topics, and every section contains a number of application examples and problems. An index of these applications on the front and back inside covers demonstrates the wide variety used in examples and exercises. Although intended for students who have completed two years of high school algebra or its equivalent, this text begins with a brief review of algebra that, if covered, will aid in preparing students for the work ahead.

Pedagogical Features

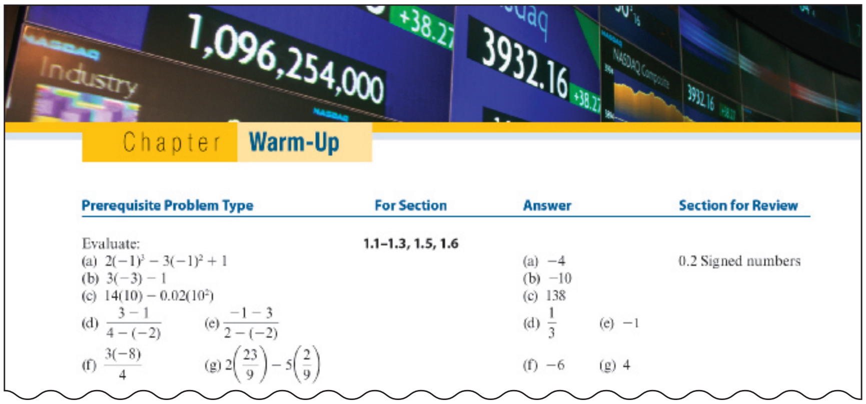
In this new edition, we have incorporated many suggestions that reflect the needs and wishes of our users, including effective pedagogical features from previous editions.

Intuitive Viewpoint. The book is written from an intuitive viewpoint, with emphasis on concepts and problem solving rather than on mathematical theory. Yet each topic is carefully developed and explained, and examples illustrate the techniques involved.

Flexibility. At different colleges and universities, the coverage and sequencing of topics may vary according to the purpose of the course and the nature of the student audience. To accommodate alternate approaches, the text has a great deal of flexibility in the order in which topics may be presented and the degree to which they may be emphasized.

Applications. We have found that integrating applied topics into the discussions and exercises helps provide motivation within the sections and demonstrates the relevance of each topic. Numerous real-life application examples and exercises represent the applicability of the mathematics, and each application problem is identified so the instructor or student can select applications that are of special interest. In addition, we have found that offering separate lessons on applied topics such as cost, revenue, and profit functions brings the preceding mathematical discussions into clear, concise focus and provides a thread of continuity as mathematical sophistication increases. There are 10 such sections throughout the book and two application-focused chapters: Chapter 4, devoted to linear programming, and Chapter 6, devoted to financial applications. Of the more than 5500 exercises in the book, more than 2000 are applied.

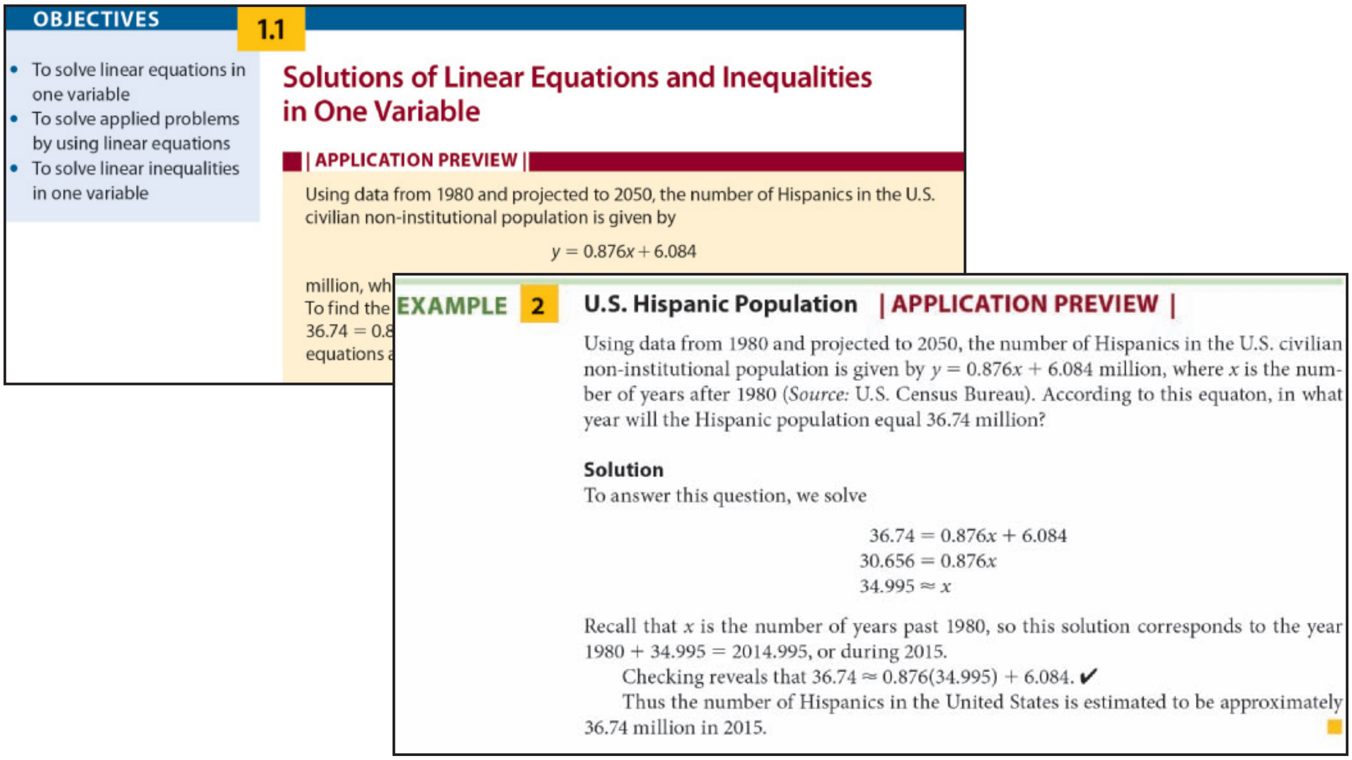
Chapter Warm-ups. With the exception of Chapter 0, a Warm-up appears at the beginning of each chapter and invites students to test themselves on the skills needed for that chapter. The Warm-up sections present many prerequisite problem types that are keyed to the appropriate sections in the upcoming chapter where those skills are needed. Students who have difficulty with any particular skill are directed to specific sections of the text for review. Instructors may also find the Warm-ups useful in creating a course syllabus that includes an appropriate scope and sequence of topics.



Chapter Warm-Up

Prerequisite Problem Type	For Section	Answer	Section for Review
Evaluate: (a) $2(-1)^3 - 3(-1)^2 + 1$ (b) $3(-3) - 1$ (c) $14(10) - 0.02(10^2)$ (d) $\frac{3-1}{4-(-2)}$ (e) $\frac{-1-3}{2-(-2)}$ (f) $\frac{3(-8)}{4}$ (g) $2\left(\frac{23}{9}\right) - 5\left(\frac{2}{9}\right)$	1.1–1.3, 1.5, 1.6	(a) -4 (b) -10 (c) 138 (d) $\frac{1}{3}$ (e) -1 (f) -6 (g) 4	0.2 Signed numbers

Application Previews and Associated Examples. Each section begins with an Application Preview that establishes the context and direction for the concepts that will be presented. Each of these Previews motivates the mathematics in the section and references a completely worked Application Preview Example appearing later in the section.



OBJECTIVES

- To solve linear equations in one variable
- To solve applied problems by using linear equations
- To solve linear inequalities in one variable

1.1 Solutions of Linear Equations and Inequalities in One Variable

| APPLICATION PREVIEW |

Using data from 1980 and projected to 2050, the number of Hispanics in the U.S. civilian non-institutional population is given by

$$y = 0.876x + 6.084$$

million, wh… To find the… 36.74 = 0.8… equations…

EXAMPLE 2 U.S. Hispanic Population | APPLICATION PREVIEW |

Using data from 1980 and projected to 2050, the number of Hispanics in the U.S. civilian non-institutional population is given by $y = 0.876x + 6.084$ million, where x is the number of years after 1980 (*Source:* U.S. Census Bureau). According to this equaton, in what year will the Hispanic population equal 36.74 million?

Solution

To answer this question, we solve

$$36.74 = 0.876x + 6.084$$
$$30.656 = 0.876x$$
$$34.995 \approx x$$

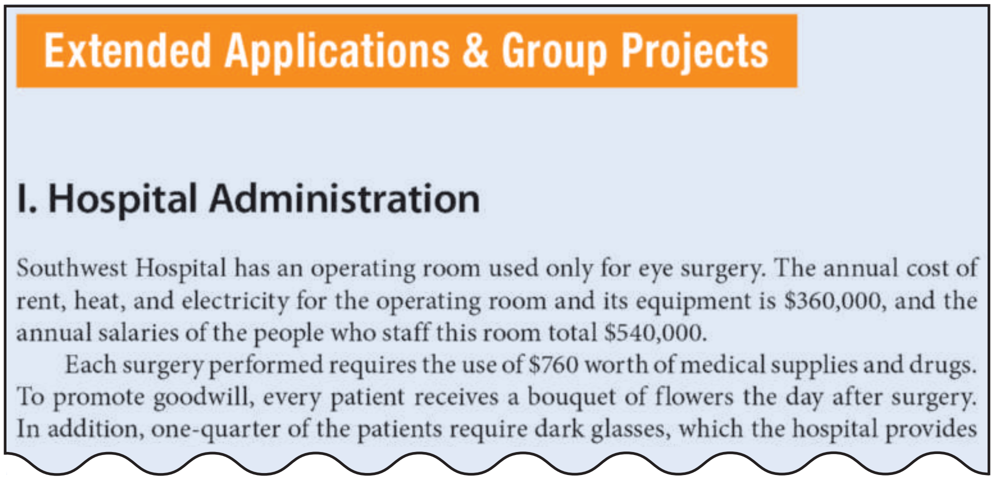
Recall that x is the number of years past 1980, so this solution corresponds to the year $1980 + 34.995 = 2014.995$, or during 2015.

Checking reveals that $36.74 \approx 0.876(34.995) + 6.084$. ✔

Thus the number of Hispanics in the United States is estimated to be approximately 36.74 million in 2015.

Comprehensive Exercise Sets. The overall variety and grading of drill and application exercises offer problems for different skill levels, and there are enough challenging problems to stimulate students in thoughtful investigations. Many exercise sets contain critical-thinking and thought-provoking multistep problems that extend students' knowledge and skills.

Extended Applications and Group Projects. Starting with Chapter 1, each chapter ends with at least two case studies, which further illustrate how mathematics can be used in business and personal decision making. In addition, many applications are cumulative in that solutions require students to combine the mathematical concepts and techniques they learned in some of the preceding chapters.


Extended Applications & Group Projects

I. Hospital Administration

Southwest Hospital has an operating room used only for eye surgery. The annual cost of rent, heat, and electricity for the operating room and its equipment is $360,000, and the annual salaries of the people who staff this room total $540,000.

Each surgery performed requires the use of $760 worth of medical supplies and drugs. To promote goodwill, every patient receives a bouquet of flowers the day after surgery. In addition, one-quarter of the patients require dark glasses, which the hospital provides

Graphical, Numerical, and Symbolic Methods. A large number of real data modeling applications are included in the examples and exercises throughout the text and are denoted by the header ***Modeling.*** Many sections include problems with functions that are modeled from real data, and some problems ask students to model functions from the data given. These problems are solved by using one or more graphical, numerical, or symbolic methods.

Graphing Calculators and Excel. Instructors differ on how they use technology in their course. The icon on the left denotes the many examples, applications, Technology Notes, Calculator Notes, and Spreadsheet Notes throughout the text where technology use is featured or appropriate. Many of these notes reference detailed step-by-step instructions in Appendix A (Graphing Calculator Guide) and Appendix B (Excel Guide) and in the Online Guide for Excel. Discussions of the use of technology are placed in subsections and examples in many sections so they can be emphasized or de-emphasized at the option of the instructor.

The discussions of graphing calculator technology highlight its most common features and uses, such as graphing, window setting, Trace, Zoom, Solver, tables, finding points of intersection, numerical derivatives, numerical integration, matrices, solving inequalities, and modeling (curve fitting). While technology never replaces the mathematics, it does supplement and extend the mathematics by providing opportunities for generalization and alternative ways of understanding, doing, and checking. Some exercises that are better worked with the use of technology—including graphing calculators and Excel—are highlighted with the technology icon. Of course, many additional exercises can benefit from the use of technology, at the option of the instructor. Technology can be used to graph functions and to discuss the generalizations, applications, and implications of problems being studied.

Excel is useful in solving problems involving linear equations; systems of equations; quadratic equations; matrices; linear programming; output comparisons of $f(x)$, $f'(x)$, and $f''(x)$; and maxima and minima of functions subject to constraints. Excel is also a useful problem-solving tool when studying the mathematics of finance in Chapter 6.

Checkpoints. The Checkpoints ask questions and pose problems within each section's discussion, allowing students to check their understanding of the skills and concepts under discussion before they proceed. Answers to these Checkpoints appear before the section exercises. Complete solutions are available on the textbook's companion web site (www.cengagebrain.com).

EXAMPLE 4 Solving an Equation for One of Two Variables

Solve $4x + 3y = 12$ for y.

Solution

No fractions or parentheses are present, so we subtract $4x$ from both sides to get only the term that contains y on one side.

$$3y = -4x + 12$$

Dividing both sides by 3 gives the solution.

$$y = -\frac{4}{3}x + 4$$

Check: $4x + 3\left(\frac{-4}{3}x + 4\right) \stackrel{?}{=} 12$

$4x + (-4x + 12) = 12$ ✔

✓ CHECKPOINT 2. Solve for y: $y - 4 = -4(x + 2)$

✓ CHECKPOINT ANSWERS

1. (a) $x = 0$ (b) $x = -63$ (c) $x = -12$
2. $y = -4x - 4$
3. $y \le 3$
4. $y < \frac{1}{2}$
5. $y \le -\frac{1}{7}$

Objective Lists. Every section begins with a brief list of objectives that outline the goals of that section for the student.

OBJECTIVES

- To solve linear equations in one variable
- To solve applied problems by using linear equations
- To solve linear inequalities in one variable

1.1 Solutions of Linear Equations and Inequalities in One Variable

| APPLICATION PREVIEW |

Using data from 1980 and projected to 2050, the number of Hispanics in the U.S.

Procedure/Example and Property/Example Tables. Appearing throughout the text, these tables aid student understanding by giving step-by-step descriptions of important procedures and properties with illustrative examples worked out beside them.

Solving a Linear Equation

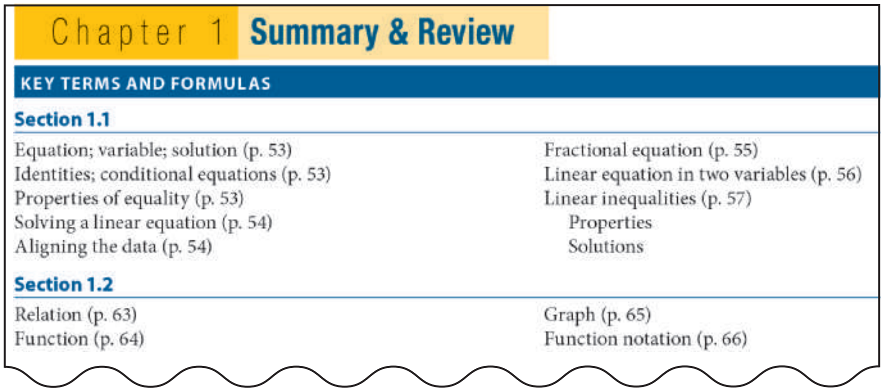
Procedure	Example
To solve a linear equation in one variable:	Solve $\frac{3x}{4} + 3 = \frac{x-1}{3}$.
1. If the equation contains fractions, multiply both sides by the least common denominator (LCD) of the fractions.	1. LCD is 12.
2. Remove any parentheses in the equation.	

Properties of Equality

Properties	Examples
Substitution Property	
The equation formed by substituting one expression for an equal expression is equivalent to the original	$3(x - 3) - \frac{1}{2}(4x - 18) = 4$ is equivalent to $3x - 9 - 2x + 9 = 4$ and to $x = 4$. We say the solution set is

Boxed Information. All important information is boxed for easy reference, and key terms are highlighted in boldface.

Key Terms and Formulas. At the end of each chapter, just before the Chapter Review Exercises, there is a section-by-section listing of that chapter's key terms and formulas, including their page references. This provides a well-organized core from which a student can build a review, both to consult while working the Review Exercises and to identify quickly any section needing additional study.


Chapter 1 Summary & Review

KEY TERMS AND FORMULAS

Section 1.1

Equation; variable; solution (p. 53)
Identities; conditional equations (p. 53)
Properties of equality (p. 53)
Solving a linear equation (p. 54)
Aligning the data (p. 54)
Fractional equation (p. 55)
Linear equation in two variables (p. 56)
Linear inequalities (p. 57)
Properties
Solutions

Section 1.2

Relation (p. 63)
Function (p. 64)
Graph (p. 65)
Function notation (p. 66)

Review Exercises and Chapter Tests. At the end of each chapter, a set of Review Exercises offers students extra practice on topics in that chapter. These reviews cover each chapter's topics in their section order, with section references, so that students get a thorough, structured review and can readily find a section for further review if difficulties occur. A Chapter Test follows each set of Review Exercises. All Chapter Tests provide a mixture of problems that do not directly mirror the order of topics found within the chapter. This organization of the Chapter Test ensures that students have a firm grasp of material in the chapter. All answers to both the Review Exercises and Chapter Tests appear in the Answers section.

Changes in the Eleventh Edition

In the eleventh Edition, we continue to offer a text characterized by complete and accurate pedagogy, mathematical precision, excellent exercise sets, numerous and varied applications, and student-friendly exposition. The most significant changes to this edition are as follows.

- Most of the real-data application examples, exercises, and Group Projects that are data based have been updated or replaced. This includes new real-data projects in Chapters 5, 9, and 11.
- The Checkpoints were redesigned and relocated to enhance their effectiveness. In addition, new Checkpoints were added as needed. Checkpoint answers appear before the section exercises. Complete solutions are available on the textbook's companion web site (www.cengagebrain.com).
- While full graphing calculator and Excel details remain in the Appendices, more details, including specific steps and screen shots, were integrated in the text.
- The Chapter Summary and Key Terms were redesigned, with page references included to improve their effectiveness.
- Section references were added to the Chapter Reviews to help Instructors assign review problems and to help students focus their review. The Chapter Tests do not contain section references in order to provide a comprehensive review in anticipation of testing.
- Specific steps for solving financial problems with graphing calculators and Excel have been added to Chapter 6 and retained in the Appendices. Because of the ease of solving financial problems with technology, the financial tables were removed from the Appendices.
- Additional applications relating to environmental issues and demographics were added.

- A discussion of significant digits was added to the modeling introduction in Chapter 2, and significant digits were integrated in modeling discussions throughout the text.
- Example solutions were made more effective and student-friendly by adding steps where needed and removing excessive detail where possible.
- Example redundancies were removed.
- Several Application Previews were streamlined to more quickly and effectively engage students.
- Exposition was streamlined wherever appropriate.

Chapter-Specific Changes

Chapter 0:

- The discussion of Properties of Real Numbers was improved.
- Formula evaluation discussion and an example were added.
- The discussion of polynomial multiplication was unified.

Chapter 1:

- Exercises involving writing equations of lines from their graphs were added.

Chapter 2:

- The discussion of polynomial functions was expanded; examples and exercises including the y-intercept were added.
- More specific instructions were added to the modeling procedures.
- Significant digits were defined, and rules for their use were discussed.
- The modeling examples and exercises were reorganized to improve grading and variety. All real-data applications involving dates were replaced or updated.

Chapter 4:

- The exercises throughout the chapter were reorganized to improve grading and eliminate redundancy.
- The Application Previews in Sections 4.1 and 4.2 were streamlined.
- The discussion and the example introducing graphical methods for solving linear programming problems were simplified.
- The introduction to the simplex method for solving linear programming problems in Section 4.3 was rewritten to shorten and simplify it while retaining the applied context of constraints and slack variables.
- The applied exercises in Section 4.5 were categorized to help instructors assign problems.

Chapter 5:

- Discussion and an example of solving logarithmic equations were added.
- A new Group Project was added.

Chapter 6:

- The use of graphing calculators and Excel in the solution of financial applications was strengthened and now includes specific steps and screen shots in the text as well as in the Appendix. The financial tables were removed from the Appendix.
- The exercises in Sections 6.3, 6.4, and 6.5 and the Chapter 6 Review that require multiple financial ideas for their solution were isolated and labeled as *Combined Applications*.
- Redundant examples and exercises were removed to streamline the discussion.
- The balance between exercises involving ordinary annuities and annuities due in Sections 6.3 and 6.4 was improved in both the labeled and the miscellaneous exercises.

- The discussion of bond pricing was improved by clarifying the exposition and adding a figure.
- A discussion of loan refinancing and an accompanying example and exercises were added.

Chapter 7:

- A new Group Project that uses Bayes' formula was added.

Chapter 8:

- The introduction to the normal distribution was streamlined and clarified.
- Discussion and exercises involving finding intervals or threshold values in a normal distribution satisfying given probability conditions and using inverse normal calculations were added.

Chapter 9:

- The Warm-up was improved, and piecewise functions were added.
- Definitions and exposition were streamlined and clarified where appropriate.
- The number of examples and exercises involving derivative calculations with variables other than x and y was increased.
- An additional real-data Group Project was added.

Chapter 10:

- Details were added to the solution steps in applied max-min examples.
- An example to illustrate all possible cases for horizontal asymptotes of rational functions was added.

Chapter 11:

- The drill exercises in the Chapter Review were rebalanced to improve grading and variety.
- A new real-data modeling Group Project was added.

Chapter 12:

- The drill exercises were improved and expanded.

Chapter 14:

- The exposition in the Test for Maximum and Minimum box as well as in examples and exercises was improved.
- The notation in the development of linear regression formulas was clarified.

Resources for the Student

Student Solutions Manual (978-1-305-10806-6)
This manual provides complete worked-out solutions to all odd-numbered exercises in the text, giving you a chance to check your answers and ensure you took the correct steps to arrive at an answer.

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Complete Solutions Manual
The Complete Solutions Manual provides worked-out solutions of all exercises in the text. In addition, it contains the solutions of the special features in the text, such as *Extended Applications and Group Projects*. This manual can be found on the Instructor Companion Site.

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Acknowledgments

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0 Algebraic Concepts

This chapter provides a brief review of the algebraic concepts that will be used throughout the text. You may be familiar with these topics, but it may be helpful to spend some time reviewing them. In addition, each chapter after this one opens with a warm-up page that identifies prerequisite skills needed for that chapter. If algebraic skills are required, the warm-up cites their coverage in this chapter. Thus you will find that this chapter is a useful reference as you study later chapters.

The topics and some representative applications studied in this chapter include the following.

SECTIONS		APPLICATIONS
0.1	**Sets** Set operations Venn diagrams	Dow Jones Industrial Average, jobs growth
0.2	**The Real Numbers** Inequalities and intervals Absolute value	Income taxes, health statistics
0.3	**Integral Exponents**	Personal income, endangered species
0.4	**Radicals and Rational Exponents** Roots and fractional exponents Operations with radicals	Richter scale, half-life
0.5	**Operations with Algebraic Expressions**	Revenue, profit
0.6	**Factoring** Common factors Factoring trinomials	Simple interest, revenue
0.7	**Algebraic Fractions** Operations Complex fractions	Average cost, advertising and sales

0.1

Sets

A **set** is a well-defined collection of objects. We may talk about a set of books, a set of dishes, a set of students, or a set of individuals with a certain blood type. There are two ways to tell what a given set contains. One way is by listing the **elements** (or **members**) of the set, in any order and usually between braces. We may say that a set A contains 1, 2, 3, and 4 by writing $A = \{1, 2, 3, 4\}$. To say that 4 is an element of set A, we write $4 \in A$. Similarly, we write $5 \notin A$ to denote that 5 is not an element of set A.

If all the elements of the set can be listed, the set is said to be a **finite set.** $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$ are examples of finite sets. When we do not wish to list all the elements of a finite set, we can use three dots to indicate the unlisted elements of the set. For example, the set of even integers from 8 to 8952, inclusive, could be written as

$$\{8, 10, 12, 14, \ldots, 8952\}$$

Since we cannot list all the elements of an **infinite set**, we use the three dots to indicate that the list continues. For example, $N = \{1, 2, 3, 4, \ldots\}$ is an infinite set. This set N is called the set of **natural numbers.**

Another way to specify the elements of a given set is by description. For example, we may write $D = \{x: x \text{ is a Ford automobile}\}$ to describe the set of all Ford automobiles. Furthermore, $F = \{y: y \text{ is an odd natural number}\}$ is read "F is the set of all y such that y is an odd natural number."

EXAMPLE 1 Describing Sets

Write the following sets in two ways.

(a) The set A of natural numbers less than 6

(b) The set B of natural numbers greater than 10

(c) The set C containing only 3

Solution

(a) $A = \{1, 2, 3, 4, 5\}$ or $A = \{x: x \text{ is a natural number less than } 6\}$

(b) $B = \{11, 12, 13, 14, \ldots\}$ or $B = \{x: x \text{ is a natural number greater than } 10\}$

(c) $C = \{3\}$ or $C = \{x: x = 3\}$ ■

A set that contains no elements is called the **empty set** or the **null set,** and it is denoted by $\varnothing$ or by $\{\ \}$. The set of living veterans of the War of 1812 is empty because there are no living veterans of that war. Thus

$$\{x: x \text{ is a living veteran of the War of 1812}\} = \varnothing$$

Special relations that may exist between two sets are defined as follows.

Relations between Sets

Definition	Example
1. Sets X and Y are **equal** if they contain the same elements.	1. If $X = \{1, 2, 3, 4\}$ and $Y = \{4, 3, 2, 1\}$, then $X = Y$.
2. A is called a **subset** of B, which is written $A \subseteq B$ if every element of A is an element of B. The empty set is a subset of every set. Each set A is a subset of itself.	2. If $A = \{1, 2, c, f\}$ and $B = \{1, 2, 3, a, b, c, f\}$, then $A \subseteq B$. Also, $\varnothing \subseteq A$, $\varnothing \subseteq B$, $A \subseteq A$, and $B \subseteq B$.
3. If C and D have no elements in common, they are called **disjoint.**	3. If $C = \{1, 2, a, b\}$ and $D = \{3, e, 5, c\}$, then C and D are disjoint.

In the discussion of particular sets, the assumption is always made that the sets under discussion are all subsets of some larger set, called the **universal set** U. The choice of the universal set depends on the problem under consideration. For example, in discussing the set of all students and the set of all female students, we may use the set of all humans as the universal set.

We may use **Venn diagrams** to illustrate the various relationships among sets. A rectangle represents the universal set, and closed figures inside the rectangle represent the sets under consideration. Figures 0.1–0.3 show such Venn diagrams.

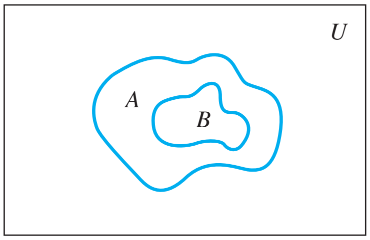


Figure 0.1
B is a subset of A; $B \subseteq A$.

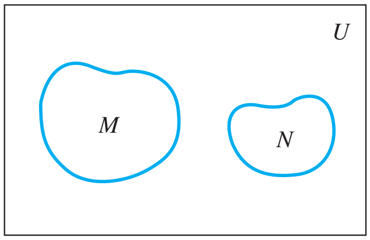


Figure 0.2
M and N are disjoint.

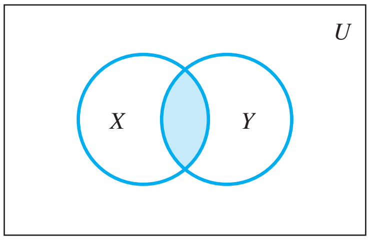


Figure 0.3
X and Y are not disjoint.

✓ CHECKPOINT

Let $A = \{2, 3, 5, 7, 11\}$, $B = \{2, 4, 6, 8, 10\}$, and $C = \{6, 10, 14, 18, 22\}$. Use these sets to answer the following.

1. (a) Of which sets is 6 an element?
   (b) Of which sets is $\{2\}$ a subset?
2. Which of the following are true?
   (a) $2 \in A$
   (b) $\{2\} \in B$
   (c) $2 \in C$
   (d) $5 \notin A$
   (e) $5 \notin B$
3. Which pair of A, B, and C is disjoint?
4. Which of $\varnothing$, A, B, and C are subsets of
   (a) the set P of all prime numbers?
   (b) the set M of all multiples of 2?
5. Which of A, B, and C is equal to $D = \{x: x = 4n + 2 \text{ for natural numbers } 1 \le n \le 5\}$?

Set Operations

The set containing the members that are common to two sets is said to be the **intersection** of the two sets.

Set Intersection

The intersection of A and B, written $A \cap B$, is defined by

$$A \cap B = \{x: x \in A \text{ and } x \in B\}$$

EXAMPLE 2 Set Intersection

(a) If $A = \{2, 3, 4, 5\}$ and $B = \{3, 5, 7, 9, 11\}$, find $A \cap B$.
(b) Which of A, B, and $A \cap B$ is a subset of A?

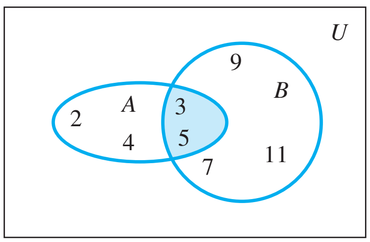


Figure 0.4

Solution

(a) $A \cap B = \{3, 5\}$ because 3 and 5 are the common elements of A and B. Figure 0.4 shows the sets and their intersection.
(b) $A \cap B$ and A are subsets of A.

The **union** of two sets is the set that contains all members of the two sets.

Set Union

The union of A and B, written $A \cup B$, is defined by

$$A \cup B = \{x: x \in A \text{ or } x \in B \text{ (or both)}\}^*$$

EXAMPLE 3 Set Union

If $X = \{a, b, c, f\}$ and $Y = \{e, f, a, b\}$, find $X \cup Y$.

Solution

$X \cup Y = \{a, b, c, e, f\}$

We can illustrate the intersection and union of two sets by the use of Venn diagrams. The shaded region in Figure 0.5 represents $A \cap B$, the intersection of A and B, and the shaded region in Figure 0.6—which consists of all parts of both circles—represents $A \cup B$.

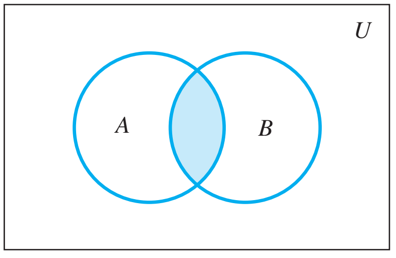


Figure 0.5
Intersection of A and B.

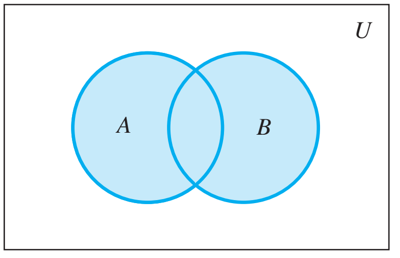


Figure 0.6
Union of A and B.

EXAMPLE 4 Set Intersection and Union

Let $A = \{x: x \text{ is a natural number less than } 6\}$ and $B = \{1, 3, 5, 7, 9, 11\}$.
(a) Find $A \cap B$.
(b) Find $A \cup B$.

Solution

Note that $A = \{1, 2, 3, 4, 5\}$.
(a) $A \cap B = \{1, 3, 5\}$
(b) $A \cup B = \{1, 2, 3, 4, 5, 7, 9, 11\}$

All elements of the universal set that are not contained in a set A form a set called the **complement** of A.

Set Complement

The complement of A, written A', is defined by

$$A' = \{x: x \in U \text{ and } x \notin A\}$$

We can use a Venn diagram to illustrate the complement of a set. The shaded region of Figure 0.7 represents A', and the *un*shaded region of Figure 0.5 represents $(A \cap B)'$.

* In mathematics, the word *or* means "one or the other or both."

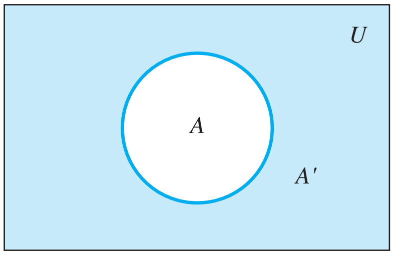


Figure 0.7

EXAMPLE 5 Operations with Sets

If U is the set of natural numbers less than 10, $A = \{1, 3, 6\}$, and $B = \{1, 6, 8, 9\}$, find the following.

(a) A'
(b) B'
(c) $(A \cap B)'$
(d) $A' \cup B'$

Solution

(a) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ so $A' = \{2, 4, 5, 7, 8, 9\}$
(b) $B' = \{2, 3, 4, 5, 7\}$
(c) $A \cap B = \{1, 6\}$ so $(A \cap B)' = \{2, 3, 4, 5, 7, 8, 9\}$
(d) $A' \cup B' = \{2, 4, 5, 7, 8, 9\} \cup \{2, 3, 4, 5, 7\} = \{2, 3, 4, 5, 7, 8, 9\}$ ■

✓ CHECKPOINT

Given the sets $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 5, 7\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$, find the following.

6. $A \cup B$
7. $B \cap C$
8. A'

EXAMPLE 6 Stocks

Suppose an investment advisor monitored several stocks for clients and on a certain day categorized 23 stocks according to whether

- their closing price on the previous day was less than \$50/share (set C)
- their price-to-earnings ratio was less than 20 (set P)
- their dividend per share was at least \$1.50 (set D).

Of these 23 stocks,

16 belonged to set P
12 belonged to set C
8 belonged to set D
3 belonged to both C and D
10 belonged to both C and P
7 belonged to both D and P
2 belonged to all three sets.

(a) How many stocks had closing prices of less than \$50 per share or price-to-earnings ratios of less than 20?
(b) How many stocks had none of the characteristics of set C, P, or D?
(c) How many stocks had only dividends per share of at least \$1.50?

Solution

We use a Venn diagram to organize the information. Note that the Venn diagram for three sets has eight separate regions (see Figure 0.8(a) on the next page). To assign numbers from our data, we must begin with some information that refers to a single region, namely that two stocks belonged to all three sets (see Figure 0.8(b)). Because the region common to all three sets is also common to any pair, we can next use the information about stocks that belonged to two of the sets (see Figure 0.8(c)). Finally, we can complete the Venn diagram (see Figure 0.8(d)).

(a) We need to add the numbers in the separate regions that lie within $C \cup P$. That is, 18 stocks closed under \$50 per share or had price-to-earnings ratios of less than 20.
(b) There are 5 stocks outside the three sets *C*, *D*, and *P*.
(c) Those stocks that had only dividends of at least \$1.50 per share are inside *D* but outside both *C* and *P*. There are no such stocks.

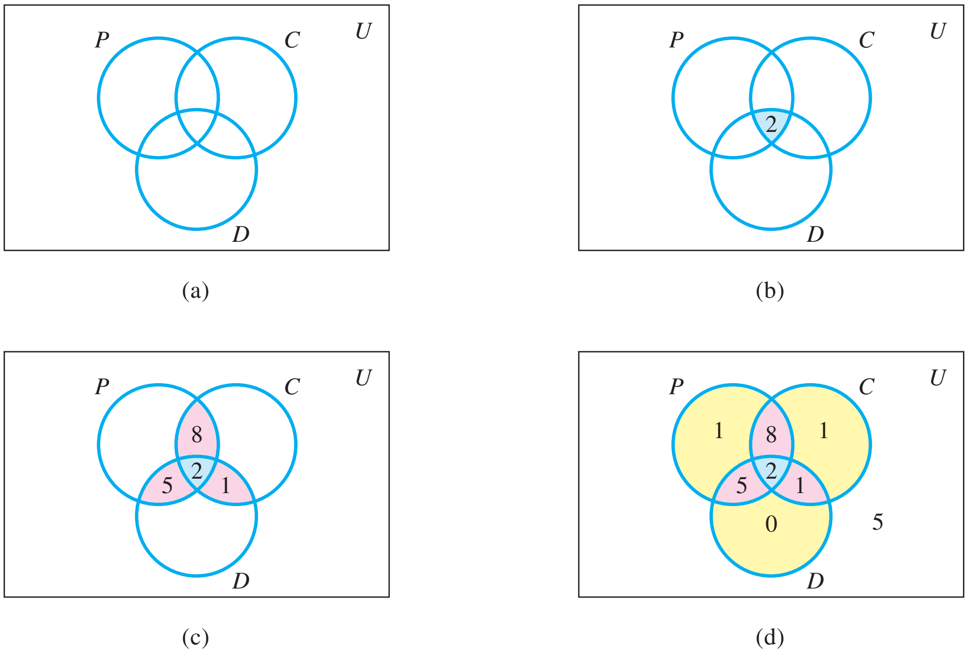


Figure 0.8

✓ CHECKPOINT ANSWERS

1. (a) *B* and *C*
   (b) *A* and *B*
2. (a) True
   (b) False
   (c) False
   (d) False
   (e) True
3. *A* and *C*
4. (a) $\varnothing$ and *A*
   (b) $\varnothing$, *B*, and *C*
5. $C = D$
6. {1, 2, 3, 5, 7, 9}
7. {5, 7}
8. {2, 4, 6, 8, 10}

| EXERCISES | 0.1

In Problems 1–4, use $\in$ or $\notin$ to indicate whether the given object is an element of the given set in the following problems.

1. 12 $\{1, 2, 3, 4, \ldots\}$
2. 5 $\{x: x$ is a natural number greater than $5\}$
3. 6 $\{x: x$ is a natural number less than $6\}$
4. 3 $\varnothing$

In Problems 5–8, write the following sets a second way.

5. $\{x: x$ is a natural number less than $8\}$
6. $\{x: x$ is a natural number greater than 6, less than $10\}$
7. $\{3, 4, 5, 6, 7\}$
8. $\{7, 8, 9, 10, \ldots\}$

In Problems 9 and 10, which of $\varnothing$, A, and B are subsets of B?

9. $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$
10. $A = \{a, b, c, d\}$ and $B = \{c, d, a, b\}$?
11. Is $A \subseteq B$ if $A = \{a, b, c, d\}$ and $B = \{a, b, d\}$?
12. Is $A \subseteq B$ if $A = \{6, 8, 10, 12\}$ and $B = \{6, 8, 10, 14, 18\}$?

In Problems 13–16, use $\subseteq$ notation to indicate which set is a subset of the other.

13. $C = \{a, b, 1, 2, 3\}$ and $D = \{a, b, 1\}$
14. $E = \{x, y, a, b\}$, $F = \{x, 1, a, y, b, 2\}$
15. $A = \{6, 8, 7, 4\}$, $B = \{8, 7, 6, 4\}$
16. $D = \{a, e, 1, 3, c\}$, $F = \{e, a, c, 1, 3\}$

In Problems 17–20, indicate whether the two sets are equal.

17. $A = \{a, b, \pi, \sqrt{3}\}$, $B = \{a, \pi, \sqrt{3}, b\}$
18. $A = \{x, g, a, b\}$, $D = \{x, a, b, y\}$
19. $D = \{x: x$ is a natural number less than $4\}$,
    $E = \{1, 2, 3, 4\}$
20. $F = \{x: x$ is a natural number greater than $6\}$,
    $G = \{7, 8, 9, \ldots\}$
21. From the following list of sets, indicate which pairs of sets are disjoint.

    $A = \{1, 2, 3, 4\}$
    $B = \{x: x$ is a natural number greater than $4\}$
    $C = \{4, 5, 6, \ldots\}$
    $D = \{1, 2, 3\}$

22. If A and B are disjoint sets, what does $A \cap B$ equal?

In Problems 23–26, find $A \cap B$.

23. $A = \{2, 3, 4, 5, 6\}$ and $B = \{4, 6, 8, 10, 12\}$
24. $A = \{a, b, c, d, e\}$ and $B = \{a, d, e, f, g, h\}$
25. $A = \varnothing$ and $B = \{x, y, a, b\}$
26. $A = \{x: x$ is a natural number less than $4\}$ and
    $B = \{3, 4, 5, 6\}$

In Problems 27–30, find $A \cup B$.

27. $A = \{1, 2, 4, 5\}$ and $B = \{2, 3, 4, 5\}$
28. $A = \{a, e, i, o, u\}$ and $B = \{a, b, c, d\}$
29. $A = \varnothing$ and $B = \{1, 2, 3, 4\}$
30. $A = \{x: x$ is a natural number greater than $5\}$ and
    $B = \{x: x$ is a natural number less than $5\}$

In Problems 31–42, let

$$\begin{aligned} \mathbf{A} &= \{1, 3, 5, 8, 7, 2\} \\ \mathbf{B} &= \{4, 3, 8, 10\} \\ \mathbf{C} &= \{2, 4, 6, 8, 10\} \end{aligned}$$

and U be the universal set of natural numbers less than 11. Find the following.

31. A'
32. B'
33. $A \cap B'$
34. $A' \cap B'$
35. $(A \cup B)'$
36. $(A \cap B)'$
37. $A' \cup B'$
38. $(A' \cup B)'$
39. $(A \cap B') \cup C'$
40. $A \cap (B' \cup C')$
41. $(A \cap B')' \cap C$
42. $A \cap (B \cup C)$

The difference of two sets, $A - B$, is defined as the set containing all elements of A except those in B. That is, $A - B = A \cap B'$. Find $A - B$ for each pair of sets in Problems 43–46 if $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

43. $A = \{1, 3, 7, 9\}$ and $B = \{3, 5, 8, 9\}$
44. $A = \{1, 2, 3, 6, 9\}$ and $B = \{1, 4, 5, 6, 7\}$
45. $A = \{2, 1, 5\}$ and $B = \{1, 2, 3, 4, 5, 6\}$
46. $A = \{1, 2, 3, 4, 5\}$ and $B = \{7, 8, 9\}$

APPLICATIONS

47. ***Dow Jones Industrial Average*** The following table shows information about yearly lows, highs, and percentage changes for the years 2000 to 2012. Let L be the set of years where the low was greater than 8000. Let H be the set of years where the high was greater than 11,000. Let C be the years when the percentage change (from low to high) exceeded 35%.
    (a) List the elements of L, H, and C.
    (b) Is any of L, H, or C a subset of one of the others (besides itself)?
    (c) Write a verbal description of C'.
    (d) Find $H' \cup C'$ and describe it in words.
    (e) Find $L' \cap C$ and describe it in words.

Dow Jones Industrial Average			
Year	Low	High	% Change
2012	12,101.46	13,610.15	12.5
2011	10,655.30	12,810.54	20.2
2010	9686.48	11,585.33	19.6
2009	6547.05	10,092.19	54.1
2008	7552.29	13,056.72	72.9
2007	12,050.41	14,164.53	17.5
2006	10,667.39	12,510.57	17.3
2005	10,012.36	10,940.50	9.3
2004	9749.99	10,854.54	11.3
2003	7524.06	10,453.92	38.9
2002	7286.27	10,635.65	46.0
2001	8235.94	11,332.92	37.6
2000	9796.03	11,722.98	19.7

Source: Dow Jones & Company

48. ***Job growth*** The number of jobs in 2000, the number projected in 2025, and the projected annual growth rate for jobs in some cities are shown in the following table. Consider the following sets.
    A = set of cities with at least 2,000,000 jobs in 2000 or projected in 2025
    B = set of cities with at least 1,500,000 jobs in 2000
    C = set of cities with projected annual growth rate of at least 2.5%
    (a) List A, B, and C (using the letters to represent the cities).
    (b) Is any of A, B, or C a subset of the other?
    (c) Find $A \cap C$ and describe the set in words.
    (d) Give a verbal description of B'.

Cities	Jobs in 2000 (thousands)	Projected Jobs in 2025 (thousands)	Annual Rates of Increase (%)
O (Orlando)	1098	2207	2.83
M (Myrtle Beach)	133	256	2.64
L (Atlanta)	2715	4893	2.38
P (Phoenix)	1953	3675	2.56
B (Boulder)	233	420	2.38

Source: Based on data from NPA Data Services, Inc.

Carbon emission controls **Suppose that the following table summarizes the opinions of various groups on the issue of carbon emission controls. Use this table for Problems 49 and 50.**

	Whites		Nonwhites		
Opinion	Rep.	Dem.	Rep.	Dem.	Total
Favor	100	250	30	200	580
Oppose	250	150	10	10	420
Total	350	400	40	210	1000

49. Identify the number of individuals in each of the following sets.
    (a) Republicans and those who favor carbon emission controls
    (b) Republicans or those who favor carbon emission controls
    (c) White Republicans or those who oppose carbon emission controls
50. Identify the number of individuals in each of the following sets.
    (a) Whites and those who oppose carbon emission controls
    (b) Whites or those who oppose carbon emission controls
    (c) Nonwhite Democrats and those who favor carbon emission controls
51. ***Languages*** A survey of 100 aides at the United Nations revealed that 65 could speak English, 60 could speak French, and 40 could speak both English and French.
    (a) Draw a Venn diagram representing the 100 aides. Use E to represent English-speaking aides and F to represent French-speaking aides.
    (b) How many aides are in $E \cap F$?
    (c) How many aides are in $E \cup F$?
    (d) How many aides are in $E \cap F'$?
52. ***Advertising*** Suppose that a survey of 100 advertisers in *U.S. News*, *These Times*, and *World* found the following.

    14 advertised in all three
    30 advertised in *These Times* and *U.S. News*
    26 advertised in *World* and *U.S. News*
    27 advertised in *World* and *These Times*
    60 advertised in *These Times*
    52 advertised in *U.S. News*
    50 advertised in *World*

    (a) Draw a Venn diagram representing the 100 advertisers.
    (b) How many advertised in none of these publications?
    (c) How many advertised only in *These Times*?
    (d) How many advertised in *U.S. News* or *These Times*?
53. ***College enrollments*** Records at a small college show the following about the enrollments of 100 first-year students in mathematics, fine arts, and economics.

    38 take math
    42 take fine arts
    20 take economics
    4 take economics and fine arts
    15 take math and economics
    9 take math and fine arts
    12 take math and economics but not fine arts

(a) How many take none of these three courses?
(b) How many take math or economics?
(c) How many take exactly one of these three courses?

54. ***Survey analysis*** In a survey of the dining preferences of 110 dormitory students at the end of the spring semester, the following facts were discovered about Adam's Lunch (AL), Pizza Tower (PT), and the Dining Hall (DH).

30 liked AL but not PT
21 liked AL only
63 liked AL
58 liked PT
27 liked DH
25 liked PT and AL but not DH
18 liked PT and DH

(a) How many liked PT or DH?
(b) How many liked all three?
(c) How many liked only DH?

55. ***Blood types*** Blood types are determined by the presence or absence of three antigens: A antigen, B antigen, and an antigen called the Rh factor. The resulting blood types are classified as follows:

type A if the A antigen is present
type B if the B antigen is present
type AB if both the A and B antigens are present
type O if neither the A nor the B antigen is present

These types are further classified as *Rh-positive* if the Rh-factor antigen is present and *Rh-negative* otherwise.
(a) Draw a Venn diagram that illustrates this classification scheme.
(b) Identify the blood type determined by each region of the Venn diagram (such as A^+ to indicate type A, Rh-positive).
(c) Use a library or another source to find what percentage of the U.S. population has each blood type.

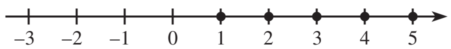
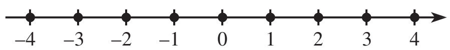
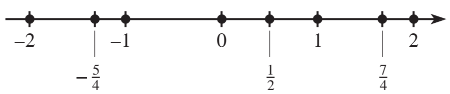
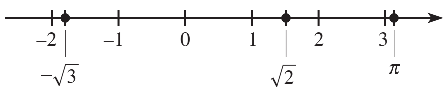
0.2

The Real Numbers

In this text we use the set of **real numbers** as the universal set. We can represent the real numbers along a line called the **real number line.** This number line is a picture, or graph, of the real numbers. Each point on the real number line corresponds to exactly one real number, and each real number can be located at exactly one point on the real number line. Thus, two real numbers are said to be equal whenever they are represented by the same point on the real number line. The equation $a = b$ (a equals b) means that the symbols a and b represent the same real number. Thus, $3 + 4 = 7$ means that $3 + 4$ and 7 represent the same number. Table 0.1 lists special subsets of the real numbers.

TABLE 0.1

SUBSETS OF THE SET OF REAL NUMBERS

	Description	Example (some elements shown)
Natural numbers	{1, 2, 3, . . .} The counting numbers.	
Integers	{. . . , −2, −1, 0, 1, 2, . . .} The natural numbers, 0, and the negatives of the natural numbers.	
Rational numbers	All numbers that can be written as the ratio of two integers, a/b, with $b \neq 0$. These numbers have decimal representations that either terminate or repeat.	
Irrational numbers	Those real numbers that *cannot* be written as the ratio of two integers. Irrational numbers have decimal representations that neither terminate nor repeat.	
Real numbers	The set containing all rational and irrational numbers (the entire number line).